AP® Calculus BC
2001 Sample Student Responses

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B1

CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3\sin(t^2)}{\cos(t^3)}$ @ $t=2$, $\frac{dy}{dx} = 15.604$

$$\boxed{(y-5) = 15.604(x-4)}$$

Work for problem 1(b)

eed= $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{\cos^2(t^3) + 9\sin^2(t^2)}$

@ $t=2$, speed = $\boxed{2.275}$

B_2

Work for problem 1(c)

$$\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt = \int_0^1 \sqrt{\cos^2(t^3)+9\sin^2(t^2)}\,dt$$

using calc

$$= \boxed{1.458}$$

Work for problem 1(d)

$x'(t) = \cos(t^3)$

$x(t) = F(t) = \int \cos(t^3)\,dt$

$$\int_2^3 \cos(t^3)\,dt = F(3) - F(2)$$

using calc

given

$-.0465 = F(3) - 4$

$\boxed{x(3) = F(3) = 3.954}$

$y'(t) = 3\sin(t^2)$

$y(t) = F(t) = \int 3\sin(t^2)$

$$\int_2^3 3\sin(t^2)\,dt = F(3) - F(2)$$

$-.0936 = F(3) - 5$

$\boxed{y(t) = F(3) = 4.906}$

Position at time $t = 3$: $\boxed{(3.954, 4.906)}$

D1

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

10:10

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx} \qquad \frac{3\sin(t^2)}{\cos(t^3)} \qquad \frac{dy}{dx} = \frac{3\sin(2)^2}{\cos(2)^3}$$

slope = 15.604

$$y - 5 = 15.604(x - 4)$$

$$\boxed{y = 15.604x - 57.417}$$

Work for problem 1(b)

$$\frac{dx}{dt} = \cos t^3 = \cos 2^3 = -.146$$

$$\frac{dy}{dt} = 3\sin t^2 = 3\sin 2^2 = -2.270$$

$$(-.146)^2 + (-2.270)^2 = s^2$$

$$s = \sqrt{(\qquad)}$$

$$\boxed{s = 2.275}$$

D2

Work for problem 1(c)

$$\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt = \int_0^1 \sqrt{(\cos t^3)^2+(3\sin t^2)^2}\,dt$$

$$\boxed{= \int_0^1 \sqrt{\cos^2 t^3 + 9\sin^2 t^2}\,dt}$$

Work for problem 1(d)

$$\cancel{x=\int \cos t^3\,dt =}$$

$$\cancel{y=3\int \sin t^2\,dt =}$$

$$\cancel{\frac{dx}{dt} = \cos t^3}$$

$$\cancel{\int dx = \int \cos t^3\,dt}$$

$$\boxed{(3.954, 4.969)}$$

at $t=2$ $x=4$ $y=5$

$\cancel{4} + \int_2^3 \cos t^3\,dt = x$ $x = 3.954$

$5 + \int_2^3 \sin t^2\,dt = y$ $y = 4.969$

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t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

A_1

Work for problem 2(a)

$$\left.\frac{dW}{dt}\right|_{t=12} \approx \left.\frac{\Delta W}{\Delta t}\right|_{9<t<15} = \frac{24-21}{9-15} = -.5°C/\text{day}$$

Work for problem 2(b)

$$\text{Average } W = \frac{\int_0^{15} W\,dt}{\Delta t} = \frac{\int_0^{15} W\,dt}{15} \approx \frac{\overset{\text{bases}}{\left(\frac{20+31}{2}+\frac{31+28}{2}+\frac{28+24}{2}+\frac{24+22}{2}+\frac{22+21}{2}\right)}\overset{\text{height}}{3}}{15}$$

$$A_{trap} = \frac{b_1+b_2}{2}\cdot h$$

$h=3$

$$A_{ve_W} = 25.1°C$$

A_2

Work for problem 2(c)

$$P'(t) = 10e^{-\frac{t}{3}} + 10t \cdot \frac{-1}{3} e^{-\frac{t}{3}}$$

$$P'(12) = 10e^{-\frac{12}{3}}\left(1 + \frac{-1}{3}t\right)$$

$$-30e^{-4} = -.5495°C/day$$

On the 12th day in the 15 day time period, the temperature of the pond will be decreasing at such a rate, that it will be getting about .5495 degrees C colder per day.

Work for problem 2(d)

$$\text{Average } P = \frac{\int_0^{15} P\,dt}{\Delta t} = \frac{\int_0^{15} 20 + 10te^{-\frac{1}{3}t}\,dt}{15}$$

$$= \frac{20t\Big|_0^{15} + 10\int_0^{15} te^{-\frac{1}{3}t}\,dt}{15}$$

$$= \frac{300 + 10\left|-3e^{-\frac{1}{3}t}(t+3)\right|_0^{15}}{15}$$

$$= \frac{300 + 86.3616}{15} = \boxed{25.757°C}$$

$$\int te^{-\frac{1}{3}t}\,dt \qquad u = t \quad v = -3e^{-\frac{1}{3}t} \qquad du = dt \quad dv = e^{-\frac{1}{3}t}d$$

$$= -3te^{-\frac{1}{3}t} + 3\int e^{-\frac{1}{3}t}\,dt$$

$$= -3te^{-\frac{1}{3}t} - 9e^{-\frac{1}{3}t} + C$$

$$= -3e^{-\frac{1}{3}t}(t+3) + C$$

C_1

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

Work for problem 2(a)

$$W'(12) \approx \frac{W(9)-W(15)}{9-15}$$

$$\approx \frac{24-21}{9-15}$$

$$W'(12) \approx -\frac{1}{2}$$

Work for problem 2(b)

$$\text{Ave } W(t) = \frac{1}{2}\cdot(3)\left(\frac{W(0)+2W(3)+2W(6)+2W(9)+2W(12)+W(15)}{15-0}\right)$$

$$\text{Ave } W(t) = \frac{1}{10}\left[20+(2\cdot31)+(2\cdot28)+(2\cdot24)+(2\cdot22)+21\right]$$

$$\text{Ave } W(t) = \frac{1}{10}(251)$$

$$\text{Ave } W(t) = 25.1\ ^\circ\text{C}$$

C_2

Work for problem 2(c)

$$P'(t) = 10e^{(-t/3)} - \frac{10}{3}te^{(-t/3)}$$

$$P'(12) = 10e^{-12/3} - (10/3)(12)\left(e^{(\frac{-12}{3})}\right)$$

$$P'(12) = 7.5495 \text{ degrees/day}$$

P'(12) is the instantaneous rate of change of degrees in celcius per day, when t=12 days.

Work for problem 2(d)

$$\text{Ave } P(t) = \frac{1}{15-0}\int_0^{15}\left[20+10te^{(-t/3)}\right]dt$$

$$\text{Ave } P(t) = \frac{1}{15}(386.362)$$

$$\text{Ave } P(t) = 25.757 \text{ °C}$$

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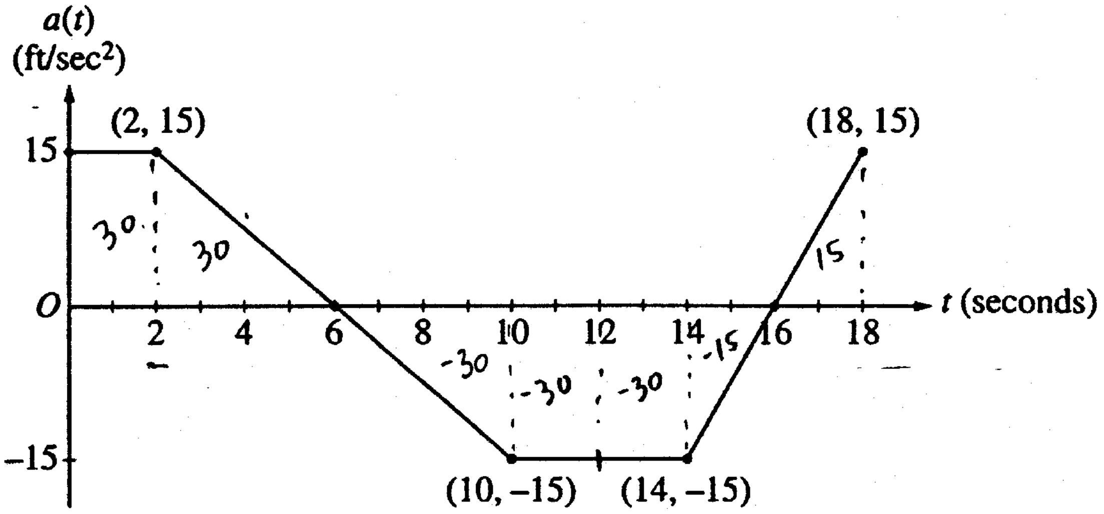
A1



Work for problem 3(a)

Yes, the velocity is increasing at t=2s because according to the graph, the car is accelerating at that time. Acceleration is positive ($a_{t=2} = 15$ ft/s²), thus the car must be getting faster.

Work for problem 3(b)

$v = \int_0^t a\,dt$

initial velocity ↓

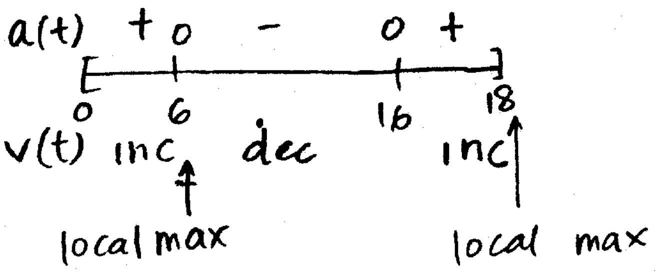
∴ velocity at time x = area under graph between t=0 and t=x +55

∴ v=55 when area under graph is zero

v=55 ft/s at t=12

A_2

Work for problem 3(c)



$V_6 = \int_0^6 a(t)\,dt + 55$

$= 60 + 55$

$= 115 \text{ ft/s}$

$V_{18} = \int_0^{18} a(t)\,dt + 55$

$= -30 + 55$

$= 25 \text{ ft/s}$

$\therefore$ absolute max velocity = 115 ft/s

at it occurs at t = 6s

Work for problem 3(d)

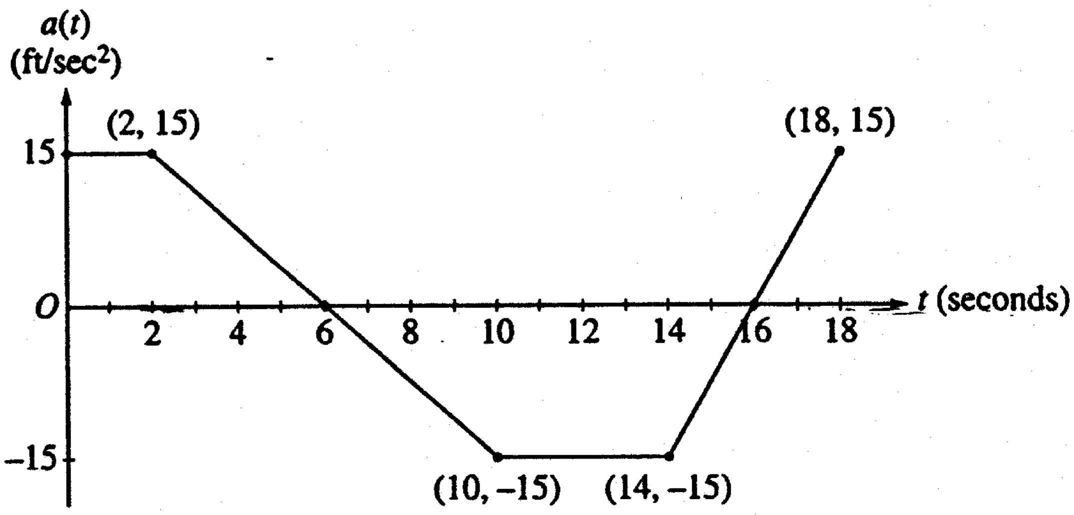
$V = \int_0^x a(t)\,dt + 55$

$-55 = \int_0^x a(t)\,dt$

for velocity to be zero, the area under the graph must be -55, which does not occur between t = 0s and t = 18s, as shown by the graph.

The lowest velocity on the interval $0 \le t \le 18$ is at t = 16, and it is 10 ft/s.

D1



Work for problem 3(a)

At $t=2$ seconds, the acceleration of the car is positive $15\frac{ft}{sec^2}$ $\therefore$

yes, the velocity of the car is increasing

Work for problem 3(b)

$v(t) = \int_0^x a(t)\,\delta t \quad + 55\frac{ft}{sec}$

$55\frac{ft}{sec.} = \int_0^x a(t)\,dt \quad + 55\frac{ft}{sec}$

$0 = \int_0^x a(t)\,dt$

from graph $\int_0^{12} a(t)\,dt = 0 \quad \therefore \quad v(12) = 55\frac{ft}{sec}$

$2(15)+\frac{1}{2}(4)(15)$

$-2(15)+\frac{1}{2}(4)(15) = 0$

$\boxed{t=12}$

D_2

Work for problem 3(c)

$v'(t) = a(t)$

from graph

$v'(t)$ pos. $[0, 6)$

$v'(t) = 0 \quad t = 6$

$v'(t)$ neg. $(6, 16)$ - -

$v'(t) = 0 \quad t = 16$

$v'(t)$ pos $(16, 18]$

A maximum occurs at $t = 6$ because the acceleration switches from positive to negative at that point and continues to be negative for an interval longer than it is positive.

$$v(t) = \int_0^x a(t)\,dt + 55\frac{ft}{sec}$$

$$v(6) = \int_0^6 a(t)\,dt + 55\frac{ft}{sec}$$

$$= 55\frac{ft}{sec} + 2sec\left(15\frac{ft}{sec^2}\right) + \frac{1}{2}(4\ sec)\left(15\frac{ft}{sec^2}\right) = \boxed{115\frac{ft}{sec}}$$

Work for problem 3(d)

$$v(t) = \int_0^x a(t)\,dt + 55\frac{ft}{sec}$$

$$0 = \int_0^x a(t)\,dt + 55\frac{ft}{sec}$$

$-55\frac{ft}{sec} \neq \int_0^x a(t)\,dt$ from graph, the $\int a(t)dt$ never equals $55\frac{ft}{sec}$

the velocity is never equal to zero

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NO CALCULATOR ALLOWED

A1

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

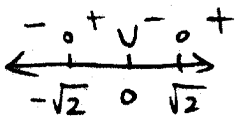
$$h'(x) = 0 = \frac{x^2-2}{x}$$

$$0 = x^2-2$$

$$x = \pm\sqrt{2}$$

∴ h has horizontal tangents at $x=\sqrt{2}$ and $x=-\sqrt{2}$

$h'(x)$



∴ h has a local minimum at $x=\sqrt{2}$ and $x=-\sqrt{2}$

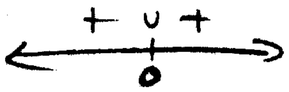
Work for problem 4(b)

$$h''(x) = \frac{2x\cdot x - (x^2-2)}{x^2}$$

$$= \frac{2x^2-x^2+2}{x^2}$$

$$= \frac{x^2+2}{x^2}$$

$h''(x)$



∴ h is concave up on the intervals $(-\infty,0)$ and $(0,\infty)$

A2

Work for problem 4(c)

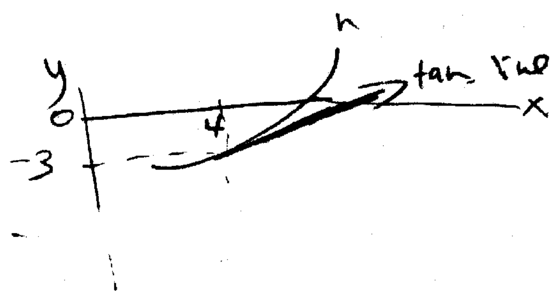
$h'(4) = \frac{4^2-2}{4} = \frac{14}{4} = \frac{7}{2}$

$h(4) = -3$

$y+3 = \frac{7}{2}(x-4)$

Work for problem 4(d)

The tangent line to the graph of h at x = 4 lies below the graph of h for x > 4 because h is concave up on the interval (0, ∞).



NO CALCULATOR ALLOWED

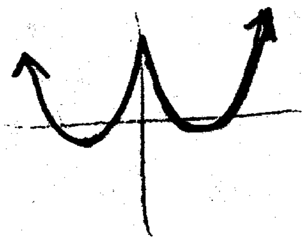
D1

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3



No calculator is allowed for these problems.

Work for problem 4(a)

$1 - \frac{2}{x^2} : x$

$x - \frac{2}{x}$

$h'(x) = 0$

$x \neq 0$ $\quad \frac{x^2-2}{x} = 0$

$x - \frac{2}{x}$

$x^2 - 2 = 0$

$x^2 = 2$

$\boxed{x = \pm\sqrt{2}}$

$h''(x) = \frac{2x^2 - x^2 + 2}{x^2}$

$h''(+\sqrt{2}) = \frac{4-2+2}{2}$

$h''(\sqrt{2}) = \frac{4}{2} = 2$

$h''(\sqrt{2})$ +

$h''(-\sqrt{2})$ +

at $+\sqrt{2}$, minimum

+ at $-\sqrt{2}$, minimum

because the second derivative is +, which means the slope is increasing

Work for problem 4(b)

$x \neq 0$

$\frac{2x^2 - x^2 + 2}{x^2} > 0$

~~2x²x~~

$x^2 + 2 > 0$

$x^2 > -2$

x^2

(h) is concave up for all values, as x^2 must always be greater than -2

D2

Work for problem 4(c)

$h(4) = -3$

$$h'(x) = \frac{x^2-2}{x}$$

$$h'(4) = \frac{16-2}{4} = \frac{14}{4} = \frac{7}{2}$$

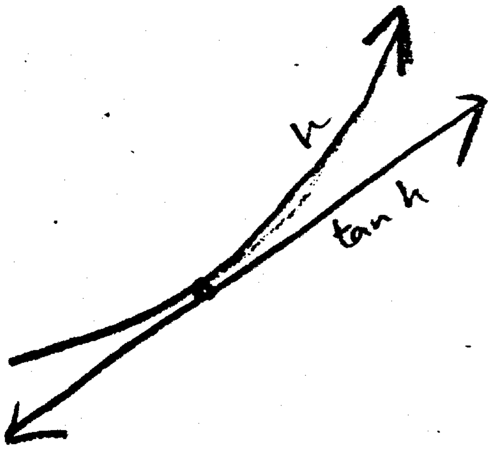
$$y+3 = \frac{7}{2}(x-4)$$

$$y = \frac{7}{2}x - 14 - 3$$

$$\boxed{y = \frac{7}{2}x - 17}$$

Work for problem 4(d)

below, because the graph is concaved up



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A1

Work for problem 5(a)

$f'(x) = -3x f(x)$

$\lim_{b\to\infty} \int_1^b -3x f(x)\, dx$

$\lim_{b\to\infty} \int_1^b f'(x)\, dx = \lim_{b\to\infty} (f(x)]_1^b$

$= \lim_{b\to\infty} f(b) - f(1)$

$= 0 - 4 = -4$

Work for problem 5(b)

$f'(x) = -3x f(x)$

Δx	.5	.5	.5
x	1	1.5	2
$f(x)$	4	-2	2.5
$f'(x)$	-12	9	

$f'(1) = -3(1)(4) = -12$

$f'(1.5) = -3(\frac{3}{2})(-2) = 9$

$f(2) \approx 2.5$

Work for problem 5(c)

A_2

$$\frac{dy}{dx} = -3xy$$

$$\frac{dy}{y} = -3x\,dx$$

$$\ln y = -\frac{3}{2}x^2 + c$$

$$y = e^{-\frac{3}{2}x^2 + c}$$

$$y = Ce^{-\frac{3}{2}x^2}$$

$$4 = Ce^{-\frac{3}{2}(1)^2}$$

$$C = \frac{4}{e^{-3/2}} = 4e^{3/2}$$

$$f(x) = y = 4e^{3/2}\left(e^{-\frac{3}{2}x^2}\right)$$

Work for problem 5(a)

D_1

a) $\int_1^{\infty} -3x\, f(x)\, dx = \lim_{6 \to \infty} \int_1^{6} -3x\, f(x)\, dx$

$= \lim_{b \to \infty} \left((-3x)^2 - (-3x)(-3) \right]_1^b$

$f = -3x$

$g = f(x)$

$g' = f'(x)$

$f' = -3$

$= \lim_{b \to \infty} \left(9x^2 - 9x \right]_1^b \Big)$

$= \lim_{b \to \infty} 9b^2 - 9b - \cancel{9+1}$

-8

$= \boxed{\infty}$

Work for problem 5(b)

$\Delta x = .5$

$\Delta y =$ slope at $f_n(x_n, y_n) \cdot \Delta$

$y_{n+\Delta x} = y_n + \Delta y$

$f(1) = 4$

① at $(1,4)$, slope $= f'(1) = -3(1)4$

$= -12$

$\Delta y = -12(.5)$

$= -6$

$y_{1.5} = 4 + -6$

$= -2 \quad \therefore \quad f(1.5) = -2$

$(1.5, -2)$

② at $(1.5, -2)$, slope $= f'(1.5) = -3(1.5)(-2)$

$= 9$

$\Delta y = 9(.5)$

$= 4.5$

$y_{2.0} = -2 + 4.5 = 2.5 \quad \therefore \quad \boxed{f(2) = 2.5}$

$(2, 2.5)$

Work for problem 5(c)

D₂

① $\frac{dy}{dx} = -3xy$

$\frac{1}{y} dy = -3x \, dx$

② $\int \frac{1}{y} \, dy = \int -3x \, dx$

③ $\ln y = -\frac{3x^2}{2} + C$

$f(1) = 4 \therefore \quad \ln 4 = -\frac{3(1)^2}{2} + c$

$\ln 4 = -\frac{3}{2} + c$

④ $c = \ln 4 + \frac{3}{2}$

⑤ $\ln y = -\frac{3x^2}{2} + \ln 4 + \frac{3}{2}$

$$\boxed{y = f(x) = e^{\left(-\frac{3x^2}{2} + \ln 4 + \frac{3}{2}\right)}}$$

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NO CALCULATOR ALLOWED

B1

Work for problem 6(a)

Ratio Test

$$L = \lim_{n\to\infty} \left| \frac{(n+2)\,x^{n+1}}{3^{n+2}} \cdot \frac{3^{n+1}}{(n+1)\,x^{n}} \right| = \lim_{n\to\infty} \left| \frac{x}{3} \cdot \frac{n+2}{n+1} \right| = \left|\frac{x}{3}\right| < 1$$

$$-1 < \frac{x}{3} < 1 \quad \therefore -3 < x < 3$$

$$x = -3: \sum_{n=0}^{\infty} \frac{(n+1)(-3)^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{(n+1)(-1)^n(3)^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{(-1)^n(n+1)}{3} \quad \text{Diverges}$$

$$x = 3: \sum_{n=0}^{\infty} \frac{(n+1)\,3^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{n+1}{3} \quad \text{Diverges}$$

$$\boxed{(-3,3)}$$

Work for problem 6(b)

$$\frac{f(x) - \frac{1}{3}}{x} = \frac{2}{3^2} + \frac{3}{3^3}x + \ldots \frac{(n+2)}{3^{n+2}}x^n$$

$$\lim_{x\to 0} \frac{f(x) - \frac{1}{3}}{x} = \frac{2}{3^2} = \boxed{\frac{2}{9}}$$

B2

Work for problem 6(c)

$$\int_0^1 f(x)\,dx = \left[\frac{1}{3}x + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \frac{x^4}{3^4} + \dots + \frac{x^n}{3^n}\right]\Bigg|_0^1$$

$$= \boxed{\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots + \frac{1}{3^n}}$$

Work for problem 6(d)

The series determined in part c is a geometric series with initial term $t_1 = \frac{1}{3}$ and a ratio $r = \frac{1}{3}$.

$$\text{Sum} = \frac{t_1}{1-r} = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \boxed{\frac{1}{2}}$$

NO CALCULATOR ALLOWED

D1

Work for problem 6(a)

$$\lim_{x\to\infty} \frac{\frac{(n+1+1)}{3^{n+1+1}}\, x^{n+1}}{\frac{(n+1)(x^n)}{3^{n+1}}} \qquad \lim_{x\to\infty} \frac{(n+2)(\cancel{x^n}\cdot x^1)}{\cancel{3^n}\cdot 3^2} \cdot \frac{\cancel{3^n}\cdot 3^1}{(n+1)(\cancel{x^n})} \qquad \Rightarrow \lim_{n\to\infty} \frac{n+\overset{1}{\cancel{2}}}{\cancel{n}+1} \cdot \frac{x}{3}$$

$$\left|\frac{x}{3}\right|<1 \qquad -1<\frac{x}{3}<1 \qquad -3<x<3$$

Check endpoints

$\sum \frac{n+1}{3^{n+1}}(-3)^n$ converges $\qquad \sum \frac{n+1}{3^{n+1}}(3^n)$ diverges

$$\boxed{-3 \le x < 3}$$

Work for problem 6(b)

$$\lim_{x\to 0} \frac{f(x)-\frac{1}{3}}{x} \Rightarrow \lim_{x\to 0} \frac{\frac{2}{3^2}x+\frac{3}{3^3}x^2+\dots+\frac{n+1}{3^{n+1}}x^n+\dots}{x}$$

$$\lim_{x\to 0} \frac{2}{3^2}+\cancel{\frac{3}{3^3}x+\dots}\xrightarrow{}0 \quad \frac{n+1}{3^{n+1}}x^{n-1}$$

$$=\frac{2}{3^2} = \boxed{\frac{2}{9}}$$

D_2

Work for problem 6(c)

$$g(x) = \frac{1}{3}x + \frac{\cancel{2}x^2}{3^2 \cdot \cancel{2}} + \frac{\cancel{3}x^3}{3^3 \cdot \cancel{3}}$$

$$g(x) = \frac{1}{3}x\Big|_0^1 + \frac{x^2}{9}\Big|_0^1 + \frac{x^3}{27}\Big|_0^1$$

$$\boxed{g(x) \quad \frac{1}{3} + \frac{1}{9} + \frac{1}{27}}$$

Work for problem 6(d)

$$S_n = \frac{a_1}{1-r} \qquad a_1 = \frac{1}{3} \qquad r = \frac{1}{3}$$

$$S_n = \frac{\frac{1}{3}}{1-\frac{1}{3}} \qquad S_n = \frac{\frac{1}{3}}{\frac{3}{3}-\frac{1}{3}} \qquad S_n = \frac{\frac{1}{3}}{\frac{2}{3}} \qquad S_n = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{2} = \boxed{\frac{1}{2}}$$